MODULE 6: RATIONAL NUMBERS, PART 3

Percents and Mixed Numbers

Just as we were very interested in powers of ten when we studied the arithmetic of whole numbers, we are also very interested in common fractions whose denominators are powers of ten. Think of how often we hear such remarks as: "On a scale of 10, I'd give it a 7." We hardly, if ever, hear: "On a scale of 37, I'd give it a 23!"

In the language of common fractions we'd think of this as 7/10.

When you take a test in school and get "80" as your grade, it does not usually mean that you got 80 correct answers on the test. For example, there might only have been 5 questions on the test and you answered 4 of them correctly. What your instructor may have decided was that at a rate of 4 out of 5, you'd have gotten 80 out of 100.

Again, in terms of common fractions:

$$\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100}$$

In other words, 4 on a scale of 5 translates into 80 on a scale of 100.

In fact, your teacher might have written the grade as "80%"

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New Vocabulary

The symbol "%" is called a *percent sign*. *We read "80%" as "80 percent"*

"per" is Latin and means "for (each)". "cent" is an abbreviation for the Latin word, "centum" which means "100". That is, "percent" means "for each 100"

In this context, 80% of a number means $\frac{80}{100}$ *of that number.*

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80 per hundred would be written as 80/100. As an abbreviation the fraction bar was placed between the two 0's and the 1 was omitted. That is, 80/100 became 80 0/0. Later 0/0 gave way to the symbol "%" to avoid any confusion.

Example 1

How much is 73% of 300?

Answer: 219

73% means "73 per 100" or $\frac{73}{100}$. Hence:

$$73\% \text{ of } 300 =$$
$$\frac{73}{100} \text{ of } 300 =$$
$$\frac{73}{100} \times \frac{300}{1} =$$
$$\frac{73 \times \cancel{300}}{\cancel{100} \times 1} =$$
$$\frac{219}{1} =$$
$$219$$

If the details seem "fuzzy" review Module 5.

In the language of common fractions:

$$\frac{73}{100} = \frac{73 \times 3}{100 \times 3} = \frac{219}{300}$$

In other words, at a rate of 73 per 100, we'd take 219 per 300. Or: 73 on a scale of 100 is equivalent to 219 on a scale of 300.

Sometimes we word Example 1 differently.

Example 2

A test consists of 300 true-false questions. A student answered 219 of these questions correctly. What percent of the questions did the student answer correctly?

Answer: 73%

In terms of equivalent common fractions we're being asked to solve:

$$\frac{219}{300} = \frac{?}{100}$$

That is, we're asking: "219 on a scale of 300 is how many on a scale of 100?"

Observing that 3 is a factor of both numerator and denominator, we have that:

$$\frac{219}{300} = \frac{\cancel{3} \times 73}{\cancel{3} \times 100} = \frac{73}{100}$$

In terms of fractional parts, we want to know how much is 219/300 of 100. We get:

$$\frac{219}{\cancel{300}} \times \frac{\cancel{100}}{1} = \frac{219}{3}$$

$$\begin{array}{r} 73 \\ 3\overline{)219} \\ -21 \\ \hline 9 \\ -9 \\ \hline 0 \end{array}$$

Under the conditions of Example 2, the teacher would mark the paper with a grade of "73%"

You should notice that 73% is a rate. We don't know the actual amount unless we know what we're taking 73% of.

Example 3

How much is 73% of 600?

Answer: 438

This time we want $\frac{73}{100}$ of 600.

Hence:

$$\frac{73}{100} \text{ of } 600 = \frac{73}{100} \times 600$$
$$= \frac{73}{1\cancel{00}} \times \frac{6\cancel{00}}{1}$$
$$= \frac{73 \times 6}{1 \times 1}$$
$$= \frac{438}{1}$$
$$= 438$$

In terms of equivalent common fractions, 219/300 and 438/600 are both equivalent to 70/100 (70%).

If we compare Examples 3 and 1, we see that in both examples we're taking 73% of an amount, but because 600 is twice 300, 73% of 600 is twice 73% of 300.

This happens a lot in real-life. If all the workers get a 10% raise, then the workers with the greatest salaries get the greatest raises

Because % is a rate, it can happen that a small percent of a large number can exceed a large percent of a small number.

Example 4

How much is 22% of 3,000?

Answer: 660

This time we want $\frac{22}{100}$ of 3,000

and this is:

$$\frac{22}{1\cancel{00}} \times \frac{30\cancel{00}}{1} = \frac{22 \times 30}{1 \times 1}$$
$$= \frac{660}{1}$$
$$= 660$$

So even though 73 is greater than 22, 22% of 3,000 is greater than either 73% of 300 or 73% of 600. *It is true, of course, that 73% of a number is greater that 22% of that same number.*

It's like saying that 73 pennies are more coins than 22 dimes, but that 22 dimes has more purchasing power.

except that any percent of is still 0.

Some percents translate into very convenient common fractions. For instance:

Example 5

Express 25% as an equivalent common fraction in lowest terms.

Answer: $25\% = \frac{1}{4}$

25% means "25 per 100". This, in turn, means "25 ÷ 100", which in turn means $\frac{25}{100}$.

But

$$\frac{25}{100} = \frac{\cancel{25} \times 1}{\cancel{25} \times 4} = \frac{1}{4}$$

This is true in general. In the language of common fractions we may replace "%" by "over 100". That is, 59% of 8,000 means: $\frac{59}{100}$ *of 8,000*

In other words, 1 on a scale of 4 is equivalent to 25 on a scale of 100.

Example 6

How much is 25% of 3,000?

Answer: 750

We want 25% of 3,000; but based on Example 5, this is the same as $\frac{1}{4}$ of 3,000. In Module 4 we learned that $\frac{1}{4}$ of 3,000 meant 3,000 ÷ 4, which is 750.

In other words, at a rate of 1 per 4, we'd get 750 per 3,000. In common fractions: $\frac{1}{4} = \frac{1 \times 750}{4 \times 750} = \frac{750}{3,000}$

Example 7

Express 20% as an equivalent common fraction in lowest terms.

Answer: $20\% = \frac{1}{5}$

20% means $\frac{20}{100}$, and:

$$\frac{20}{100} = \frac{1 \times \cancel{20}}{5 \times \cancel{20}} = \frac{1}{5}$$

1 on a scale of 5 is equivalent to 20 on a scale of 100.

Example 8

How much is 20% of 3,000?

Answer: 600

Based on Example 7, 20% of 3,000 is the same as $\frac{1}{5}$ of 3,000; and this is the same as $3,000 \div 5$, or 600.

We don't have to know that 20% = 1/5 to do this example. We could simply compute 20/100 of 3,000 directly.

Examples 6 and 8 show us a convenient way to check Example 4. Namely 22% of 3,000 is greater than 20% of 3,000 but less than 25% of 3,000. Hence 22% of 3,000 must be between 600 and 750. Since 660 is between 600 and 750, we see that our answer in Example 4 is at least plausible.

For example, had we "lost" a 0 in Example 4 and obtained 66 as our answer; we'd know we made a mistake because 66 is not between 600 and 750.

While it is convenient in many ways to deal with denominators that are powers of ten, there are some drawbacks as well. For example, suppose on a test that had 10 questions a student did 7 correctly and also part of an 8th. It would be fair to give the student more than 7 but less than 8 on a scale of 10. The problem is that there are no whole numbers between 7 and 8. *Thus, if we insisted on marking on a scale of ten, we'd want to have numbers that were more than 7 but less than 8.* This will lead us to the notion of mixed numbers.

If we didn't insist on a scale of 10, we could use the facts that:
$\frac{7}{10} = \frac{70}{100}$ *and* $\frac{8}{10} = \frac{80}{100}$
So on a scale of 100; 71, 72, 73, 74, 75, 76, 77, 78, and 79 all describe numbers that are between 7 and 8 on a scale of 10.

If we wanted to give the student 7 points and half of another point, we could say that the student received $(7 + \frac{1}{2})$ points. Rather than write $7 + \frac{1}{2}$ we omit the plus sign and write the $\frac{1}{2}$ immediately to the right of the 7; that is, $7\frac{1}{2}$ --which we read as "seven and one-half".

Vocabulary

The sum of a whole number and a common fraction less than 1 is called a **mixed number**.

For example, $8 + \frac{1}{3}$ *is called a mixed number. We abbreviate it by writing* $8\frac{1}{3}$*, which we read as "eight and one-third".*

Example 9

Write $7\frac{1}{2}$ as a common fraction in lowest terms.

Answer: $\frac{15}{2}$

By definition:

$$7\frac{1}{2} = 7 + \frac{1}{2} = \frac{7}{1} + \frac{1}{2} = \frac{14}{2} + \frac{1}{2} = \frac{15}{2}$$

Once we get to this step, the rest simply involves our previous study of common fractions.

$$\frac{7}{1} = \frac{7 \times 2}{1 \times 2} = \frac{14}{2}$$

(For example, 14 half-dollars equals 7 dollars)

What we did here was to recognize that 1 is 2 halves. Hence 7 is 2 halves, 7 times; or 14 halves. If the denominator had been 3 we would have viewed 1 as 3 thirds.

Example 10

Write $5\frac{2}{3}$ as a common fraction in lowest terms.

Answer: $\frac{17}{3}$

$$5\frac{2}{3} = 5 + \frac{2}{3} = \frac{5}{1} + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$$

If you look at what we did in the last two examples, you may see a pattern for converting

mixed numbers into common fractions. Namely:

We're given a mixed number — *for example* $8\frac{3}{7}$

<u>*Step 1*</u>

We multiply the whole number by the denominator of the fractional part. — *in this case,* $8 \times 7 = 56$

<u>*Step 2*</u>

Add the answer to Step 1 to the numerator of the fractional part. — *in this case,* $56 + 3 = 59$

<u>*Step 3*</u>

Write the answer to Step 2 over the denominator of the fractional part. — *in this case,* $\frac{59}{7}$

These three steps summarize the steps:

$$
\begin{aligned}
8\tfrac{3}{7} &= 8 + \frac{3}{7} \\
&= \frac{8}{1} + \frac{3}{7} \\
&= \frac{56}{7} + \frac{3}{7} \\
&= \frac{59}{7}
\end{aligned}
$$

<u>Example 11</u>

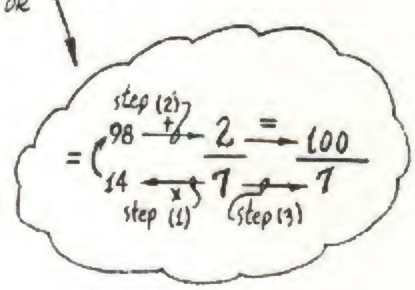
Write $14\frac{2}{7}$ as a common fraction in lowest terms.

Answer: $\frac{100}{7}$

We first multiply 14 by 7 to get 98. We then add 2 to get 100; and, finally, we put 100 over 7 to get $\frac{100}{7}$.

$$
\left\{
\begin{aligned}
14 &= \frac{14}{1} = \frac{14 \times 7}{1 \times 7} = \frac{98}{7} \\
14\tfrac{2}{7} &= 14 + \frac{2}{7} \\
&= \frac{98}{7} + \frac{2}{7} \\
&= \frac{100}{7}
\end{aligned}
\right.
$$

OR



Once we know how to write a mixed number as a common fraction, we can apply the rules of the previous modules. For example:

Example 12

Find the product of $7\frac{1}{2}$ and 4.

Method 1

Write $7\frac{1}{2}$ as $\frac{15}{2}$. Then:

$$
\begin{aligned}
7\frac{1}{2} \times 4 &= \frac{15}{2} \times 4 \\
&= \frac{15}{2} \times \frac{4}{1} \\
&= \frac{60}{2} \\
&= 30
\end{aligned}
$$

Method 2

$7\frac{1}{2}$ means $7 + \frac{1}{2}$. Therefore:

$7\frac{1}{2} \times 4 = (7 + \frac{1}{2}) \times 4.$

Now use the distributive property to rewrite $(7 + \frac{1}{2}) \times 4$ as

$$
\begin{array}{ccccc}
(7 \times 4) & + & (\frac{1}{2} \times 4) & = \\
28 & + & 2 & = \\
& 30 & &
\end{array}
$$

Answer: 30

Check: $7\frac{1}{2}$ is half-way between 7 and 8. So $7\frac{1}{2} \times 4$ should be half-way between 7×4 (28) and 8×4 (32). 30 is halfway between 28 and 32.

Equivalently, $\frac{15}{2}$ of 4 means we divide 4 by 2 to get 2; and we then multiply 2 by 15 to get 30.

The result may seem more natural in terms of money. For example, at $7.50 per pound, 4 pounds would cost $30.

Example 13

Write $7\frac{1}{2}\%$ as an equivalent common fraction in lowest terms.

$7\frac{1}{2}\%$ means $7\frac{1}{2}$ per 100 or $7\frac{1}{2} \div 100$

We then go through the following steps:

$$
\begin{aligned}
7\frac{1}{2} \div 100 &= \frac{15}{2} \div \frac{100}{1} \\
&= \frac{15}{2} \times \frac{1}{100} \\
&= \frac{15}{200} \\
&= \frac{3 \times \cancel{5}}{40 \times \cancel{5}} \\
&= \frac{3}{40}
\end{aligned}
$$

Answer: $\frac{3}{40}$

In terms of multiples:

$$
\begin{array}{rl}
& 7\frac{1}{2} \text{ per } 100 \\
+ & 7\frac{1}{2} \text{ per } 100 \\
\hline
& 15 \text{ per } 200 \text{ or } \frac{15}{200}
\end{array}
$$

The important point is that once we get

$$\frac{15}{2} \div 100$$

we proceed as in Module 5.

We see problems like those in Example 13 in banking. A bank may feel that it wants to give more than a 7% interest rate but less than 8%. So we often see such rates as $7\frac{1}{2}\%$ or $7\frac{2}{3}\%$ and so on. Somehow or other, to most people $7\frac{1}{2}\%$ is more abstract than saying that for each \$40 you invest, you make \$3 in interest.

That is, 3 per 40 is often easier for us to grasp than $7\frac{1}{2}$ per 100; or $7\frac{1}{2}\%$

Sometimes we start with a common fraction and write the answer as a percent. This, too, often forces us to deal with mixed numbers.

Example 14

Write $\frac{1}{3}$ as an equivalent percent.

Answer: $\frac{1}{3} = 33\frac{1}{3}\%$

That is, 1 on a scale of 3 translates into $33\frac{1}{3}$ on a scale of 100.

We want $\frac{1}{3}$ of an amount. On a scale of 100 the amount is 100. That is, we think of "the whole" as being 100%.

In this context we want $\frac{1}{3}$ of 100%.

We already know that:

$$\begin{aligned}\frac{1}{3} \text{ of } 100 &= \frac{1}{3} \times 100\\ &= \frac{1}{3} \times \frac{100}{1}\\ &= \frac{100}{3}\end{aligned}$$

We could leave the answer as $\frac{100}{3}$, but as we shall note after the solution of this example, it is more convenient to rewrite the answer as a mixed number. To do this we proceed as we did in Module 5 to get:

When the numerator of a fraction is at least as great as the denominator, we call the fraction "improper". It is often convenient to rewrite "improper" fractions as mixed numbers.

$$\frac{100}{3} \qquad \begin{array}{r} 33 \\ 3\overline{)100} \\ -\ 9 \\ \hline 10 \\ -\ \ 9 \\ \hline 1 \end{array}$$

If this were Module 3, we'd write the quotient as 33R1, but now that we have introduced mixed numbers we use the fact that the remainder means that we have 1 of the 3 it would take to make another unit.

More generally, we write the remainder over the divisor and thus convert the quotient into a mixed number.

Therefore, we write the answer as $33\frac{1}{3}$.

Note

One advantage of writing $33\frac{1}{3}$ rather than $\frac{100}{3}$ is that $33\frac{1}{3}$ suggests at once that the number is between 33 and 34.

Example 15

Write $\frac{453}{7}$ as a mixed number.

By long division, we have:

$$\frac{453}{7} \qquad \begin{array}{r} 6\ 4\ \text{R5} \\ 7\,\overline{)4\ 5\ 3} \\ -4\ 2 \\ \hline -\ 3\ 3 \\ 2\ 8 \\ \hline 5 \end{array}$$

and since there are now 7 sevenths to a unit, we rewrite the remainder as $\frac{5}{7}$ to get $64\frac{5}{7}$.

Answer: $64\frac{5}{7}$

Check:
$$64\frac{5}{7} = 64 + \frac{5}{7} = \frac{64}{1} + \frac{5}{7} = \frac{64 \times 7}{1 \times 7} + \frac{5}{7} = \frac{448}{7} + \frac{5}{7} = \frac{453}{7}$$

As a practical example, suppose we had 453 books and could put 7 books into 1 carton. It would take 64 full cartons plus a 65th carton with 5 of the 7 books it took to fill it to store the books.

That is, 64 cartons @ 7 books per carton is 64 X 7 or 448 books, plus the other 5 books accounts for the 453 books.

Note

The denominator is important. To say that the remainder is 5 isn't enough. We often want to know: 5 out of <u>how many</u>?

For example $\frac{15}{7}$ isn't the same as $\frac{7}{3}$, but in terms of long division both would be written as: 2 R1. The proper way is to write $2\frac{1}{7}$ and $2\frac{1}{3}$

Example 16

Write $\frac{6}{13}$ as a percent.

Answer: $\frac{6}{13} = 46\frac{2}{13}\ \%$

In this case we want $\frac{6}{13}$ of the whole, or $\frac{6}{13}$ of 100%. Therefore we have:

$$\frac{6}{13} \text{ of } 100(\%) = \frac{6}{13} \times 100\ (\%)$$
$$= \frac{6}{13} \times \frac{100}{1}\ (\%)$$
$$= \frac{600}{13}\ (\%)$$

$$\frac{600}{13} \qquad \begin{array}{r} 46 \text{ R2} \\ 13\overline{)600} \\ -52 \\ \hline 80 \\ -78 \\ \hline 2 \end{array} \qquad \text{or } 46\frac{2}{13}\ (\%)$$

Rough Check

A rate of 6 out of 13 is a little less than half (6 out of 12 would be half).

Since 50% represents half, we expect 6/13 to represent a "little" less than 50%.

$46\frac{2}{13}\%$ is thus plausible.

In concluding our treatment of percents, it is important to point out that for some rates, even 1% is dangerous. There are certain toxins where even 1 part per million can be dangerous. For this reason we often encounter fractional parts of 1%.

Example 17

Write $\frac{3}{5}$ of 1% as a common fraction in lowest terms.

Answer: $\frac{3}{500}$

1% means 1 per 100 or $\frac{1}{100}$. Therefore $\frac{3}{5}$ of 1% means $\frac{3}{5}$ of $\frac{1}{100}$, or $\frac{3 \times 1}{5 \times 100}$ or $\frac{3}{500}$

In other words, a rate of $\frac{3}{5}$ of 1% translates into 3 on a scale of 500.

Rather than write $\frac{3}{5}$ of 1% we often write simply:

$$\frac{3}{5}\%$$

Caution

Do not confuse $\frac{3}{5}$ of 1% with $\frac{3}{5}$ of the whole. $\frac{3}{5}$ means we take 3 of each 5, $\frac{3}{5}$ of 1% is even less than 1%, which is only 1 out of each 100.

Read $\frac{3}{5}\%$ as $\frac{3}{5}$ per 100; or $\frac{3}{5} \div 100$

Example 18

How much is $\frac{7}{100}\%$ of 60,000?

Answer: 42

1% of 60,000 is 600 (that is, 60,000 ÷ 100) and 7/100 % is much less than 1%.

$\frac{7}{100}\%$ means :

$$\frac{7}{100} \text{ per } 100 = \frac{7}{100} \div 100 = \frac{7}{100} \div \frac{100}{1} = \frac{7}{100} \times \frac{1}{100} = \frac{7 \times 1}{100 \times 100} = \frac{7}{10,000}$$

Remember that $\frac{7}{100}$ % means $\frac{7}{100}$ of 1%

That is, $\frac{7}{100}\%$ is the same rate as 7 per 10,000

Hence $\frac{7}{100}\%$ of 60,000 means

$\frac{7}{10,000}$ of 60,000 or

$\frac{7}{\cancel{10,000}} \times \cancel{60,000}$ or 42

At a rate of $\frac{7}{100}$ per 100 all we'd take is 42 out of 60,000. Don't confuse this with 7/100 of 60,000; which is 7% of 60,000. The percent after 7/100 is very important.

Additional percent problems are left for the Self-Test. For now we'd like to spend a little time discussing the arithmetic of mixed numbers.

A Basic Strategy

In Modules 3 and 4 we learned to deal with common fractions. Hence given any mixed number problem, we can:

(1) Translate it into a common fraction problem.

(2) Do the resulting common fraction problem.

(3) Translate the answer into a mixed number.

We've learned how to convert mixed numbers to common fractions.

We learned this in Modules 3 and 4.

which we've also learned to do in this module.

Example 19

Write $6\frac{1}{2} \times 2\frac{3}{4}$ as a mixed number.

Answer: $17\frac{7}{8}$

$$6\frac{1}{2} = \frac{(6 \times 2) + 1}{2}$$

$$= \frac{13}{2}$$

$$2\frac{3}{4} = \frac{(2 \times 4) + 3}{4}$$

$$= \frac{11}{4}$$

This is Step 1--We're translating the problem into the language of common fractions

Hence:

$$6\frac{1}{2} \times 2\frac{3}{4} = \frac{13}{2} \times \frac{11}{4}$$

$$= \frac{13 \times 11}{2 \times 4}$$

$$= \frac{143}{8}$$

Now we're solving the resulting common fraction problem.

Finally:

$$\begin{array}{r} 17 \text{ R}7 \\ 8\overline{)143} \\ -8 \\ \hline 63 \\ -56 \\ \hline 7 \end{array} = 17\frac{7}{8}$$

And finally, we're translating the answer into a mixed number.

CAUTION

$6\frac{1}{2} \times 2\frac{3}{4}$ is greater than $6\frac{1}{2} \times 2$.

because $2\frac{3}{4}$ is more than 2

But $6\frac{1}{2} \times 2$ is 13. Hence the answer to Example 19 must be more than 13. Notice that if you had multiplied the whole numbers ($6 \times 2 = 12$) and the fractions ($\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$) and wrote $12\frac{3}{8}$ as the answer, it couldn't possibly be correct.

For example, at \$6.50 per pound, just 2 pounds would already cost \$13.

In the long run, it isn't enough for a procedure to "seem" right. Either it agrees with reality or it doesn't.

In doing Example 19 we see how mathematics resembles a foreign language. When you're learning a foreign language and someone asks you a question in that language, you tend to translate the question into your native tongue. Then you answer the question

in your own language. Finally you translate the answer into the foreign language.

In this context, common fractions are our native tongue. Mixed numbers are then the foreign language. So we translate from mixed numbers into common fractions, solve the resulting common fraction problem, and then translate the answer into mixed numbers.

You become bi-lingual when you hear the question in the foreign language and answer it immediately in the foreign language without first translating it into your native tongue.

In this context you might have preferred to write $6\frac{1}{2} \times 2\frac{3}{4}$ *as* $(6 + \frac{1}{2}) \times (2 + \frac{3}{4})$ *and then use the distributive property to get the answer. We shall emphasize the various approaches in the Self-Test but for now we want you to see how we can translate rational number problems into the language of common fractions.*

It's often helpful to be able to visualize any problem in terms of a specific approach. Learning each new problem as a different "trick" is often confusing--and it often tends to hide the structure of what's happening.

<u>Example 20</u>

How much is $6\frac{1}{2} : 2\frac{3}{4}$?

Proceeding as we did in Example 19, we see that

$$6\frac{1}{2} \div 2\frac{3}{4} = \frac{13}{2} \div \frac{11}{4}$$
$$= \frac{13}{2} \times \frac{4}{11}$$
$$= \frac{52}{22}$$
$$= \frac{2 \times 26}{2 \times 11}$$

Answer: $2\frac{4}{11}$

$6\frac{1}{2}$ *rounds off to 7 while* $2\frac{3}{4}$ *rounds off to 3. So the the quotient rounds off to* $7 \div 3$ *or* $2\frac{1}{3}$. *Hence* $2\frac{4}{11}$ *is at least a plausible answer.*

$$= \frac{26}{11} \qquad 11\overline{)26}\ \ 2\ R4 \text{ or } 2\frac{4}{11}$$

$$\begin{array}{r} 2 \\ 11\overline{)2\,6} \\ -\underline{2\,2} \\ 4 \end{array}$$

Example 21

Write $6\frac{1}{2} + 2\frac{3}{4}$ as a mixed number.

Answer: $9\frac{1}{4}$

Method 1

$$6\frac{1}{2} + 2\frac{3}{4} = \frac{13}{2} + \frac{11}{4}$$

$$= \frac{13 \times 2}{2 \times 2} + \frac{11}{4}$$

Remember that to add common fractions we need a common denominator.

$$= \frac{26}{4} + \frac{11}{4}$$

$$= \frac{37}{4} \qquad \begin{array}{r} 9 \\ 4\overline{)3\,7} \\ -\underline{3\,6} \\ 1 \end{array} \quad R1 = 9\frac{1}{4}$$

Method 2

We can use the associative and commutative properties of addition:

$$6\frac{1}{2} + 2\frac{3}{4} = (6 + \frac{1}{2}) + (2 + \frac{3}{4})$$

$$= (6 + 2) + (\frac{1}{2} + \frac{3}{4})$$

$$= 8 + (\frac{2}{4} + \frac{3}{4})$$

$$= 8 + \frac{5}{4}$$

$$= 8 + 1\frac{1}{4}$$

$$= 8 + (1 + \frac{1}{4})$$

$$= (8 + 1) + \frac{1}{4}$$

$$= 9 + \frac{1}{4}$$

$$= 9\frac{1}{4}$$

Because only addition is involved in this problem, we can add the whole numbers and the fractions separately But this isn't self-evident. Indeed, the same process didn't apply to multiplication. That's why it's so important to understand what you're doing--and why!

As usual, there's only one right answer, but more than one right method.

Example 22

Write $6\frac{1}{2} - 2\frac{3}{4}$ as a mixed number.

Answer: $3\frac{3}{4}$

Again we can write:

$$6\frac{1}{2} - 2\frac{3}{4} = \frac{13}{2} - \frac{11}{4}$$

$$= \frac{26}{4} - \frac{11}{4}$$

$$= \frac{15}{4} \qquad \begin{array}{r} 3 \text{ R3 or } 3\frac{3}{4} \\ 4\,\overline{)\,1\;5} \\ -\underline{1\;2} \\ 3 \end{array}$$

If we wanted to avoid translating into common fractions, we could write:

$$\begin{array}{r} 6\ \frac{1}{2} \\ -\quad 2\ \frac{3}{4} \\ \hline \end{array} \quad \textit{or} \quad \begin{array}{r} 6\ \frac{2}{4} \\ -\ 2\ \frac{3}{4} \\ \hline \end{array}$$

We can't take $\frac{3}{4}$ from $\frac{2}{4}$ so we have to exchange 1 for 4 fourths.

$$6\frac{2}{4} = 6 + \frac{2}{4}$$

$$= (5 + 1) + \frac{2}{4}$$

$$= 5 + (1 + \frac{2}{4})$$

$$= 5 + (\frac{4}{4} + \frac{2}{4})$$

$$= 5 + \frac{6}{4}$$

$$= 5\frac{6}{4}$$

Don't *write* $6\frac{2}{4} = \overset{5}{\cancel{6}}\ \frac{{}^{1}2}{4}$

We're not exchanging ten 4ths for 1 but rather four 4ths. For example, you get 4 quarters for \$1-- not 10 quarters.

Hence:

$$\begin{array}{r} 6\frac{1}{2} \\ -\ 2\frac{3}{4} \end{array} \text{ means } \begin{array}{r} 5\frac{6}{4} \\ -\ 2\frac{3}{4} \\ \hline 3\frac{3}{4} \end{array}$$

In any event this completes our treatment of percents and mixed numbers for now. In the next module we'll complete our treatment of rational numbers.

The key point of the next module will be centered

around the idea that there are times when we need a scale of a power of ten other than a hundred. For example, depending on the degree of accuracy needed in a problem, we might want parts per 1,000 or parts per 10,000 and so on.

This notion leads to the idea of decimal fractions. The treatment of decimal fractions completes the various languages we use in the study of rational numbers.